КОМПЬЮТЕРНЫЕ НАУКИ, ПРИБОСТРОЕНИЕ И АВТОМАТИЗАЦИЯ
КОМПЬЮТЕРЛІК ҒЫЛЫМДАР, АСПАП ЖАСАУ ЖӘНЕ АВТОМАТТАНДЫРУ
COMPUTER SCIENCE, INSTRUMENTATION AND AUTOMATION

UDC 004.94
IRSTI 20.01
https://doi.org/10.53364/24138614_2025_37_2_16

A.E. Abdualiyev[1], G.K Ibragim[2], G.K.Sembina[1], B.E. Ishanova[3*]
[1]International Information Technology University, Almaty, Kazakhstan
[2]Kazakh National Research Technical University named after K. Satbayev, Almaty, Kazakhstan
[3]Zhumabek Akhmetovich Tashenev University, Shymkent, Kazakhstan

[3]*E-mail: dellivine@mail.ru**

OPTIMIZATION OF REGIONAL BUDGET ALLOCATION THROUGH A NON-ZERO-SUM GAME MODEL

Abstract. *In this paper, a non-zero-sum game model for budget allocation is developed that takes into account the interests of three participants: citizens of the region (through voting), maslikhats (through taking into account the strategic goals of regional development) and businesses (focused on return on investment). In contrast to conventional linear models, this method uses intricate utility functions to better capture stakeholder conflicting interests and decision-making behaviors. By capturing diminishing marginal gains for citizens, the logarithmic utility function makes sure that more investments in a sector result in gradually less perceived advantages.*

The model is implemented using numerical optimization techniques, specifically the interior-point algorithm, which effectively handles nonlinear constraints and ensures a globally balanced budget distribution. The optimization process required 40 iterations, progressively improving the objective function, with the final total utility reaching 9.5681×10^6*. The results validate the proposed model's ability to allocate resources equitably and efficiently while maintaining alignment with regional priorities and economic considerations.*

Furthermore, the study highlights the practical implications of game-theoretic models in budget allocation, demonstrating how strategic interactions between stakeholders can shape more transparent and rational decision-making processes. The results indicate that this approach can be expanded for dynamic planning and strategic modeling, allowing for future enhancements incorporating adaptive strategies, Nash equilibrium analysis, and real-time adjustments based on changing socio-economic conditions. The proposed model significantly enhances the efficiency, fairness, and transparency of budget allocation, making it a promising tool for decision-makers in public finance and policy planning.

Keywords*: budget, information process modeling, optimization, game model, non-zero-sum model, interior point algorithm, utility function, Nash equilibrium.*

Introduction

The distribution of the budget in districts and cities is carried out by local executive bodies – maslikhats [1], [2]. Maslikhats are district representative bodies, whose members are elected by the population of various administrative-territorial units by direct vote for a term of 5 years. Since the topic of the article is related to the distribution of regional funds in the region, several literary sources related to the promotion of budget funds and their receipt in the maslikhat were studied.

Representatives of local executive bodies rarely attend regular maslikhat meetings these days. These delegates travel to certain trouble spots to familiarize themselves with the current state of affairs and examine the information gathered for the upcoming meeting. Another element causing the public's trust in official bodies to erode is the opaque process used to distribute funds. Effective financial resource management is particularly crucial for cities and districts, where budget funds must be distributed to best meet the needs of the populace in a variety of areas of activity (AA), such as education, healthcare, transportation, infrastructure, technology, culture, and the environment.

One of the most important tasks for improving the effectiveness of money use and meeting the needs of citizens is allocating the budget among regions and sectors. Conflicts of interest between residents, maslikhats, and companies frequently accompany this process in Kazakhstan.

The intricacy of these relationships is overlooked by conventional methods like equal distribution or administrative decisions. This article proposes a non-zero-sum game model that:

– takes into account the interests of citizens through voting;
– satisfies the strategic goals of maslikhats;
– maximizes the return on investment for business.

The main difference of the proposed model is the use of complex utility functions reflecting diminishing returns, strategic priorities and economic benefits.

Materials and methods

The non-zero-sum method refers to a class of game theory problems in which the payoffs of players are not related by a linear relationship, as in zero-sum games [3].

Here the sum of all players' winnings can be positive or negative:

$$\sum_{i=1}^{n} U_i \neq 0 \tag{1}$$

where U_i – i-th player's win, n – total count of players.

Let there be many players:

$$N=\{1,2,\ldots, n\} \tag{2}$$

Each player $i \in N$ have:

1. many strategies: S_1- available player actions.
2. Utility function: U_i: $S_1 \cdot S_2 \cdot \ldots . S_n$, which reflects the winnings of each player depending on the strategies of all participants.

The overall strategy of the game is a combination of the strategies of all players:

$$s = (s_1, s_2, \ldots, s_n),\ s_i \in S_i \tag{3}$$

The strategy space is defined as the cartesian product:

$$S= S_1 \cdot S_2 \cdot \ldots . S_n \tag{4}$$

Utility function - each player seeks to maximize his utility function:

$$U_i(s)= U_i(s_1, s_2, \ldots, s_n) \tag{5}$$

where $s_i \in S_i$ – i-th player's *strategy*, s – set of player strategies.

Utility functions can be different for each player, allowing their individual goals to be taken into account. Examples:

– linear utility function: $U_i = a_i \cdot x_i + b_i$, where x_i – player actions;
– nonlinear utility function: U_i=log (x_i+1).

A non-zero-sum game involves finding a Nash equilibrium, which is defined as a set of strategies $s^* \in S_i$, in which no player can increase his utility by unilaterally changing strategy [3], [4]:

$$U_i(s_i^*, s_{-i}^*) \geq U_i(s_i^*, s_{-i}^*),\ \forall s_i \in S_i, \forall i \in N \tag{6}$$

where: s_i^*- optimal strategy of player i, s_{-i}^* - optimal strategies of all other players except i.

Often, problems in non-zero-sum games are accompanied by constraints, for example:

$$g_j(s) \leq 0, \qquad k = 1, \dots, k$$

where $g_j(s)$ – constraint functions that impose conditions on players' strategies.

Basic steps in solving non-zero-sum games [5]:

– defining a set of strategies: establish the permissible actions of players;

– assigning utility functions: determine how one player's strategy affects his utility and the utility of other players;

– finding a Nash equilibrium: use analytical or numerical methods to find s^*;

– optimization: solve the problem of maximizing utility functions, given constraints.

The non-zero-sum approach has a number of benefits. It is realistic in the first place because it considers how the interests of the actors are interdependent. Second, because of its adaptability, it can be used to model both competitive and cooperative scenarios. Finally, this method allows for balance analysis, helping to find optimal strategies for all participants. In addition to game theory and the non-zero-sum technique, our study includes numerical optimization methods that answer the stated goal of maximizing the utility function.

In this paper, the optimization problem is formulated as follows:

$$\max_{b_1, b_2 \dots b_m} U_{common} = U_{citizens} + U_{\text{maslikhats}} + U_{business} \tag{8}$$

where b_i- budget allocated for i's AA, U_{common} – the total utility function of all participants.

The problem is solved taking into account the following restrictions:

– budget amount:

$$\sum_{i=1}^{m} b_i = B_{total} \tag{9}$$

– limitations for each sector:

$$L_i \leq b_i \leq U_i, \forall i$$

The *fmincon* function, which is incorporated into the MATLAB application program and is intended to minimize (or maximum) nonlinear functions with both linear and nonlinear constraints, is utilized to solve the problem [6].

The effectiveness of the suggested multi-criteria optimization model was assessed by comparing two alternative methods from previous work: the linear "level balance" model and the linear programming model with maximum citizen satisfaction functions [7].

The objective function is written as -U_{common}, because *fmincon* minimizes functions, it turns out instead of maximization U_{common}, it is necessary to minimize disutility.

The initial value (b_{o}) is a uniform budget distribution :

$$(b_{\text{o}}) = \frac{B_{total}}{m} \tag{10}$$

where (B_{total})- total budget for distribution, m - number of AA's between which the budget is distributed.

Justification of the reasons for using this mathematical library in this work:

1. support nonlinear functions: the objective function in this paper includes nonlinear components such as logarithms (log $((b_1 + 1))$ and fractions.

2. Limitations: *fmincon* library supports inequalities (9).

3. The library functions allow finding solutions even for complex multidimensional problems, minimizing the risk of numerical errors.

fmincon can employ a variety of optimization strategies, contingent on the setup. Interior Point is the best fit for our problem since it solves it by gradually getting closer to the ideal point while staying inside the feasible zone [8].

Interior Point's exceptional efficiency in handling enormous problem scales and nonlinear constraints is an advantage. Interior Point is more adept at handling intricate restrictions and converges to the ideal solution more quickly than Sequential Quadratic Programming (SQP), which necessitates more precise gradient estimations. Additionally, this approach is more reliable for handling massive volumes of data than Active Set.

The fundamental idea is that the algorithm begins at a point inside the feasible zone and works its way through limitations. Barrier functions that prohibit leaving the viable domain are used to manage constraints. The constraint region contains the original solution. By gradually lowering the value of the barrier functions, the algorithm gets closer to the ideal solution. When all constraints are taken into consideration, the final solution matches the global (or local) minimum [9].

The strategic goals set by maslikhats show how important it is to finance different economic AA's. These goals take into account the long-term tasks of social and economic development of the region. Strategic goals are expressed as shares s_i, where $s_i \in [0,1]$, and the sum of all shares is equal to 1:

$$\sum_{i=1}^{m} s_i = 1 \tag{11}$$

These strategic goals are used in the utility function of maslikhats, for comparison with the actual budget distribution of b_i. If the distribution b_i deviates significantly from the strategic goal s_i, this reduces the usefulness of maslikhats:

$$U_{maslikhat} = \sum_{i=1}^{m} (s_i - \frac{(b_i)}{B_{total}})^2 \tag{12}$$

Significant differences between the strategy and the actual distribution are penalized by this function. During optimization, the *fmincon* algorithm will try to minimize deviations, bringing them closer together $(b_i/(B_{total})$ with s_i.

From a business standpoint, ROI (Return on Investment) indicates how successful investments in a specific AA's are. A higher economic return on investment is indicated by a high ROI. The ROI of the i -th activity area is denoted as r_i. The business utility function is calculated as follows:

$$U_{\text{business}} = \sum_{i=1}^{m} \frac{r_i \cdot b_i}{1 + \alpha \cdot b_i} \tag{13}$$

where: r_i- ROI of i-th areas of activity, α- coefficient of diminishing returns.

Without α, if the ROI (r_i) for a sector is high, a business could invest in it infinitely, which does not correspond to real economic conditions. In actuality, though, the impact of investments diminishes over time; initially, they yield the greatest value, but as budgets grow, their efficacy declines.

Since the firm is focused on maximizing returns, sectors with high r_i attract more investment, which is why ROI is included in the model. To avoid imbalance, the decreasing utility function, on the other hand, restricts excessive investment in one sector. ROI is incorporated into the objective function during optimization, which gives the model realism: the company aims to boost financing for lucrative industries, while the influence of diminishing returns stabilizes the budget allocation.

Results and discussion

The dataset used was from official sources [10], such as the National Bureau of Statistics of the Republic of Kazakhstan, regional budgets for a certain period [11].

As an example of the model's operation, data from 4 regions of the Almaty region for 2021 were taken (due to the completeness and availability of the necessary information). The data on citizens' votes are presented in Table 1, where the rows correspond to the regions and the columns correspond to the AA:

Table 1 – Voting data

Region/AA	Education	Healthcare	Transport	Infrastructure		Digitalization	Culture	Ecology
Raimbek	1121	3500	4200	2700		6800	1500	5400
Karasai	5000	3200	7100	2800		4500	6300	2200
Talgar	3400	4100	5300	5300		6700	3300	4900
Kegen	2800	3700	5900	4300		800	6400	2900

Total budget (B_{total}) of four regions for 2021 = 42,656,543,000 tenge.

Strategic goals and ROI for AA’s (approximate data):

– strategic goals (maslikhats): [0.2,0.25,0.3,0.1,0.05,0.05,0.05];

– ROI (business): [0.15,0.25,0.35,0.1,0.05,0.05,0.05].

Participants of the game:

– vote for the sectors that are prioritized by citizens (player 1). The objective is to enhance budgetary satisfaction;

– maslikhats (player 2): consider the region's strategic objectives. The objective is to reduce budgetary deviations from strategic objectives;

– player 3's business is centered on AAs' return on investment (ROI). Objective: optimize financial return.

In order to properly compare the preferences of the people in various locations while allocating the budget, the votes of the residents must be normalized [12]. Votes' absolute values are influenced by population size and voter turnout, which can result in funding disparities: areas with higher densities will get more money just because they received more votes. By eliminating this effect, normalization ensures a fair distribution of resources by allowing citizens' preferences to be considered proportionately rather than in absolute quantities.

Normalization [13], which transforms absolute vote numbers into fractions so that each vote contributes the same amount regardless of the region, can be used to solve this issue:

$$W_{i,j} = \frac{V_{i,j}}{\sum_{i=1}^{n}\sum_{j=1}^{m} V_{i,j}} \tag{14}$$

where $W_{i,j}$ – normalized share of votes for region i and industry j, $V_{i,j}$- votes of citizens of the districts.

Citizens' happiness with each extra budget unit improves more slowly when funding for activity area increases due to diminishing returns. As basic needs are met, the first 10 million tenge for healthcare substantially raises satisfaction levels; however, an additional 10 million tenge (from 10 to 20 million) no longer has the same significant effect as needs become saturated. This illustrates the reality that needs in one AA (such as healthcare) can be largely met after a certain threshold. As a result, the "gain" in utility for each additional million is decreasing.

The logarithmic utility function for citizens reflects diminishing returns:

$$U_{citizens} = \sum_{i=1}^{m} \log(b_i + 1) \cdot w_i \tag{15}$$

where w_i – sector weight (based on votes).

This is in line with reality, where basic necessities are satisfied before more significant investments are made [14].

Since the return on investment in one industry is declining, the logarithmic function prevents the entire budget from being devoted to that area.

It also reflects the fact that, in practice, citizen happiness rises as spending grows, but then falls once it reaches the most basic level. Table 2 displays a comparison with different utility functions:

Table 2 - Comparative analysis of the utility function

Utility function	Formula	Peculiarities
Linear	$U = w \cdot b$	Utility grows proportionally to the budget, does not take into account diminishing returns.
Logarithmic	$U = w \cdot \log(b+1)$	Takes into account diminishing returns, does not become negative.
Quadratic	$U = w \cdot b^2$	Unrealistically growing utility for large b

The degree to which the actual budget distribution by sector aligns with the set strategic goals determines how helpful maslikhats are. Maslikhats work to reduce the gap between the budget allotted and the portion of the money required to accomplish their objectives.

Because it increases the impact of substantial errors, the quadratic function is used to penalize significant departures from the goals [15]:

$$U_{\text{maslikhat}} = \sum_{i=1}^{m} (s_i - \frac{b_i}{B_{total}})^2 \tag{16}$$

In this formula, the difference divergence expresses the difference between the strategic goal s_i and the actual budget share $\frac{b_i}{B_{total}}$. If the allocated budget matches the goals s_i, the penalty is 0, otherwise, if the deviation $s_i - \frac{b_i}{B_{total}}$ increases by 2 times, the penalty is increased by 4 times. It is also taken into account that a negative value indicates a deviation from the goals, which reduces the usefulness.

This formula uses the quadratic function since it is sensitive to deviations; even slight deviations have little effect, allowing for a fair amount of allocation flexibility. The harsher penalties for large deviations encourage budget optimization in these industries. The return on investment (ROI) from funding each AA is reflected in the model's commercial utility. But just as in real life, the impact of an investment does not increase indefinitely in proportion to the amount of money invested. Every consecutive unit of investment yields less benefit than the one before it due to a diminishing return impact.

This effect is modeled by the diminishing returns function, which demonstrates that when a corporation invests excessively in one AA, its economic efficiency declines:

$$U_{business} = \sum_{i=1}^{m} \frac{r_i \cdot b_i}{1 + \alpha \cdot b_i} \tag{17}$$

Utility grows almost linearly at first as the budget is increased, but eventually it slows down. This is because basic necessities are met first, and subsequent investments have a diminishing impact. The rate of this slowing is determined by the coefficient α; a small value results in a little decline in return, while a large value causes utility to rapidly plateau. Thus, the influence reaches its maximum even with limitless funding.

Consequently, our model's objective function integrates the utilities of all parties involved (people, maslikhats, and enterprises) while taking their interests into consideration:

$$U_{common} = \max_{b_1, b_2 \ldots, b_m} \sum_{i=1}^{m} \log(b_i + 1) \cdot w_i - \sum_{i=1}^{m} (s_i - \frac{b_i}{B_{total}})^2 + \sum_{i=1}^{m} \frac{r_i \cdot b_i}{1 + \alpha \cdot b_i} \tag{18}$$

subject to the following limitations:

$$- \quad \sum_{i=1}^{m} b_i = B_{total}$$
$$- L_i \leq b_i \leq U_i, \forall\, i$$

Given the work's connection to game theory issues, it is impossible to overlook the core idea of game theory - Nash equilibrium. Assuming that other players' strategies don't change, it depicts the situation in which each player selects the best course of action.

Since the problem is cooperative in nature, Nash equilibrium was not included in the model. A shared aim function that incorporates the objectives of all participants was developed rather than modeling each player's strategy independently. As a result, the model is more like to a cooperative game than to traditional game theory.

Furthermore, the implementation of Nash equilibrium is challenging due to its computing complexity. Each player's strategy must be specified, equilibrium must be found repeatedly using techniques like Best Response Dynamics or Monte Carlo, and sophisticated numerical techniques that need more resources than global optimization must be used.

As a result of the model's operation, the following results were obtained (figure 1):

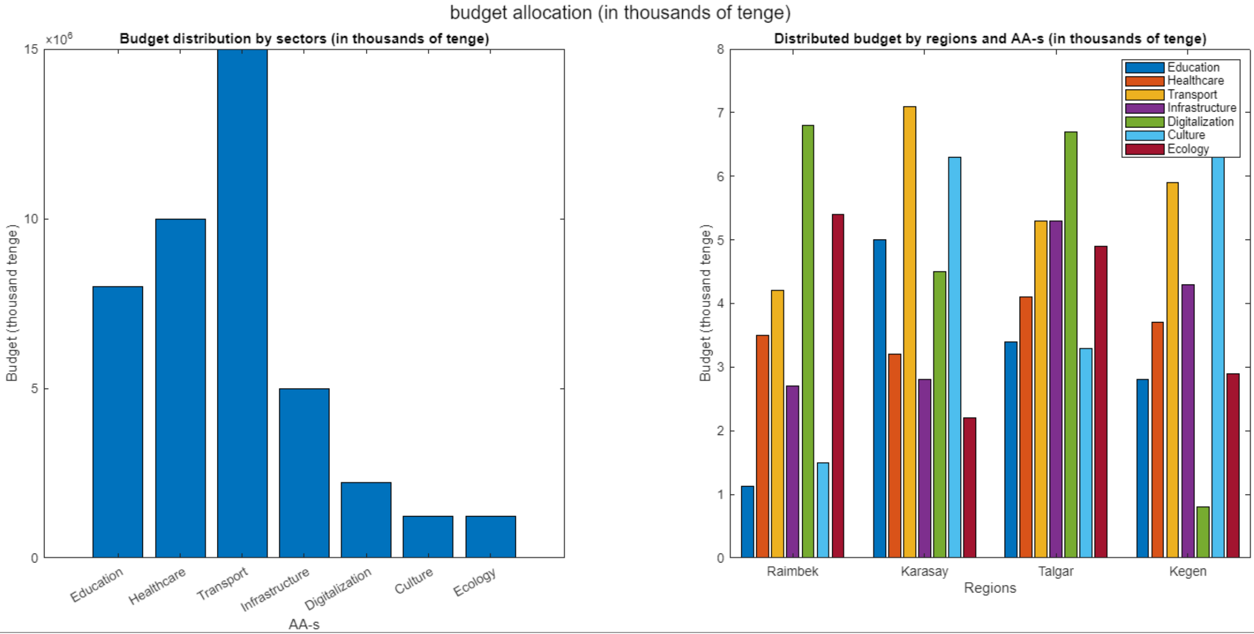

Figure 1 – the result of budget distribution by AA's and regions

The numerical results of the optimal budget distribution for the AA's are given in Table 3:

Table 3 – distributed budget for AA's:

Activity areas	Budget (thousand tenge)
Education	8000000.0
Healthcare	9999999.95
Transport	14999999.95
Infrastructure	5000000.0
Digitalization	2218479.64
Culture	1218462.67
Ecology	1219600.78

The numerical results of the optimal budget distribution by regions and their AA's are given in Table 4:

Table 4 – distributed budget by regions and their AA's (in thousands of tenge):

Region	Education	Healthcare	Transport	Infrastructure	Digitalization	Culture	Ecology
Raimbek	1121.0	3500.0	4200.0	2700.0	6800.0	1500.0	5400.0
Karasai	5000.0	3200.0	7100.0	2800.0	4500.0	6300.0	2200.0
Talgar	3400.0	4100.0	5300.0	5300.0	6700.0	3300.0	4900.0
Kegen	2800.0	3700.0	5900.0	4300.0	800.0	6400.0	2900.0

Of the overall budget (42.66 billion tenge), 35.2% went to transportation, 23.4% to healthcare, 18.8% to education, 11.7% to infrastructure, 5.2% to digitalization, 2.9% to culture, and 2.9% to ecological. At the regional level, Kegen received 3.1%, Raiymbek 3.3%, Talgar 5.2%, and Karasai district 5.7%. The strategic aims of maslikhats and the proportionate consideration of citizens' votes account for the variations in the allocation by sectors and regions. Due to their long-term return on investment, some industries (like digitalization) received a reduced proportion, but optimization corrected the funding to avoid imbalances. Stricter normalization and dynamic modeling of the effects of investments on future citizen preferences are two potentialimprovements, although the results produced are in line with the logic of equitable budget distribution.

As a result of the fmincon library operation, the data of the operation of the interior point optimization algorithm are output (figure 2):

```
Command Window
>> untitl22ed
Initial point X0 is not between bounds LB and UB;
FMINCON shifted X0 to strictly satisfy the bounds.

                                      First-order      Norm of
 Iter F-count         f(x)  Feasibility   optimality         step
    0       8  -5.389330e+06    1.238e+07    1.723e-01
    1      16  -5.389331e+06    1.238e+07    1.620e-01    1.845e+00
    2      24  -5.389332e+06    1.238e+07    1.620e-01    2.136e+00
    3      32  -5.389334e+06    1.238e+07    1.620e-01    5.586e+00
    4      40  -5.389338e+06    1.238e+07    1.620e-01    8.967e+00
    5      48  -5.389376e+06    1.237e+07    1.620e-01    8.615e+01
    6      56  -7.180411e+06    5.438e+06    1.620e-01    4.048e+06
    7      64  -7.189345e+06    5.403e+06    1.620e-01    2.024e+04
    8      72  -7.189389e+06    5.403e+06    1.620e-01    1.012e+02
    9      80  -7.189391e+06    5.403e+06    1.620e-01    3.119e+00
   10      88  -7.189394e+06    5.403e+06    1.620e-01    7.754e+00
   11      96  -7.189396e+06    5.403e+06    1.620e-01    6.004e+00
   12     104  -7.189433e+06    5.403e+06    1.620e-01    8.725e+01
   13     112  -7.190473e+06    5.400e+06    1.620e-01    2.466e+03
   14     120  -7.381895e+06    4.761e+06    1.620e-01    4.540e+05
   15     128  -7.485822e+06    4.414e+06    1.620e-01    2.466e+05
   16     136  -8.239390e+06    1.897e+06    1.620e-01    1.791e+06
   17     144  -8.243153e+06    1.884e+06    1.620e-01    8.954e+03
   18     152  -8.243172e+06    1.884e+06    1.620e-01    4.477e+01
   19     160  -8.243173e+06    1.884e+06    1.620e-01    5.081e+00
   20     168  -8.243183e+06    1.884e+06    1.620e-01    2.924e+01
   21     176  -8.243185e+06    1.884e+06    1.620e-01    7.334e+00
   22     184  -8.243186e+06    1.884e+06    1.620e-01    2.123e+00
   23     192  -8.243191e+06    1.884e+06    1.620e-01    1.448e+01
   24     200  -8.244077e+06    1.880e+06    1.620e-01    2.593e+03

   24     200  -8.244077e+06    1.880e+06    1.620e-01    2.593e+03
   25     208  -8.244644e+06    1.878e+06    1.620e-01    1.659e+03
   26     216  -8.244648e+06    1.878e+06    1.620e-01    1.108e+01
   27     224  -8.244656e+06    1.878e+06    1.620e-01    3.009e+01
   28     232  -8.244687e+06    1.878e+06    1.620e-01    1.219e+02
   29     240  -8.244697e+06    1.878e+06    1.620e-01    2.666e+01
   30     248  -8.244757e+06    1.878e+06    1.620e-01    2.051e+02

                                      First-order      Norm of
 Iter F-count         f(x)  Feasibility   optimality         step
   31     256  -8.245400e+06    1.877e+06    1.620e-01    2.217e+03
   32     264  -8.249660e+06    1.872e+06    1.620e-01    1.471e+04
   33     272  -9.540070e+06    4.397e+05    1.619e-01    4.476e+06
   34     280  -9.546492e+06    4.326e+05    1.619e-01    2.238e+04
   35     288  -9.546524e+06    4.325e+05    1.619e-01    1.119e+02
   36     296  -9.546525e+06    4.325e+05    3.447e-02    2.326e+00
   37     304  -9.546525e+06    4.325e+05    1.734e-02    2.343e+00
   38     312  -9.546525e+06    4.325e+05    1.682e-02    3.749e+00
   39     320  -9.568087e+06    7.451e-09    1.540e-02    2.497e+05
   40     328  -9.568087e+06    7.451e-09    1.540e-02    1.504e-04

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.
```

Figure 2 – Output data of the interior point optimization algorithm

The initial guess (X0) was outside the predetermined bounds, as indicated by the initial point error (Initial point X0 is not between bounds LB and UB). The original location was automatically modified by the algorithm to satisfy the restrictions.

The columns provide information about the optimization progress:

– Iter - iteration number.
– F-count - number of function calls.
– f(x) - current value of the objective function.
– Feasibility - how well the current solution satisfies the constraints.
– First-order optimality - quality indicator of the current solution.
– Norm of step - step length at this iteration.

At the beginning, the objective function f(x) was -5.3893e+06, and the constraints were far from being satisfied (1.238e+07). As the iterations increased, *f(x)* began to decrease, and the constraints became more satisfactory.

Key points of the optimization process:

At iteration 6, there was a significant jump in the objective function from -5.3893e+06 to -7.1804e+06, indicating that a more balanced distribution had been achieved.

At iteration 16, f(x) reached -8.2394e+06, and the constraints were significantly reduced (1.897e+06).

At iteration 33, there was a sharp jump to -9.5401e+06, indicating that the model had improved dramatically after a significant budget adjustment.

At iteration 39, the constraints were almost zero (7.451e-09), indicating that the solution satisfied all conditions.

The final result (Local minimum possible. Constraints satisfied) indicates that the optimization process was completed, since the step size became smaller than the threshold, all constraints were satisfied.

A total utility of 9.5681×10^6 was ensured by budget optimization, which allowed for a balanced evaluation of the interests of citizens, the strategic aims of maslikhats, and the economic efficiency of sectors. Transportation received the largest share of financing (35.2%), followed by healthcare (23.4%) and education (18.8%), all of which are critical to both economic growth and quality of life. Education is in the lead because of its long-term influence on the development of human capital, healthcare because of demographic preferences and public policy, and transportation because of its high return on investment and strategic significance. Due to their reduced investment appeal, infrastructure (11.7%), ecology (2.9%), and culture (2.9%) received an average amount of funding. Because digitalization requires large initial investments and has long-term effects, it received the least amount of money (5.2%).

The areas with the highest funding were Karasay and Talgar because of their large populations and thriving economies. Perhaps as a result of lower citizen support and a lower corporate return on investment, Raimbek district received less money. Due to maslikhat priorities, culture in Kegen district obtained a relatively large budget (6400 thousand tenge), but digitalization earned a comparatively small budget (800 thousand tenge).

During the 40 iterations of the optimization, the goal function stabilized and the most efficient use of the restricted resources was reallocated. The primary financing categories were established by Maslikhat priorities and business return on investment; nevertheless, minor adjustments to the criteria may have an impact on the redistribution of funds between digital technologies and infrastructure.

By accounting for players' dynamic interactions and strategic changes, Nash equilibrium could enhance the model. Depending on the budgetary allocation, businesses could raise their investments in the most lucrative industries, and citizens could alter their preferences.

Conclusion

For budget allocation, the suggested non-zero-sum game model made it possible to consider the interests of three important parties: corporations, maslikhats, and residents. Realistic modeling of these groups' behavior was achieved by the employment of intricate utility functions, and a balanced distribution of cash was made possible using numerical optimization techniques.

According to the findings, the largest funding allocations went to healthcare (9,999,999.95 thousand tenge), education (8,000,000.00 thousand tenge), and transportation (14,999,999.95 thousand tenge). These industries receive the greatest support from corporations and citizens and are highly significant on both a social and economic level. Infrastructure, ecology, and culture received average support, but digitization received the smallest budget because of its poorer return on investment and strategic importance.

According to the budget distribution by region, the districts of Talgar and Karasai received the highest funds, which is in line with their strategic priority and high voter turnout. Due to disparities in ROI and maslikhat priority, Raimbek and Kegen districts received relatively smaller amounts.

The interior-point method was used for optimization, allowing nonlinear constraints to be taken into consideration. After 40 iterations of optimization, the final utility was 9.5681×10^6, confirming the efficacy of the suggested strategy.

As a result, the established model has proven to be applicable to budget planning issues, providing a mechanism for equitable and financially sound distribution. In the future, dynamic elements and player strategy interactions via Nash equilibrium can be included to the model.

References

1. Ministry of Justice of the Republic of Kazakhstan, Law of the Republic of Kazakhstan dated January 23, 2001 No. 148-II. About local government and self-government in the Republic of Kazakhstan// [Electronic resource]: URL: https://online.zakon.kz/Document/?doc_id=1021546. Date of application: 1.12.2024.

2. Information and legal system of regulatory legal acts of the Republic of Kazakhstan Institute of Legislation and Legal Information of the Ministry of Justice of the Republic of Kazakhstan // [Electronic resource]: URL: https://adilet.zan.kz/rus/docs/V04D0001624. Date of application: 4.12.2024.

3. Starr, A. & Ho, Y.-C. (1969). Non-Zero-Sum Differential Games. Journal of Optimization Theory and Applications. 3. 184-206. 10.1007/BF00929443.

4. Z. Wu, Y. Mu, X. Yang and G. Chen, "Nash Equilibrium Solving for General Non-Zero-Sum Games from Non-Convergent Dynamics of Fictitious Play," 2024 43rd Chinese Control Conference (CCC), Kunming, China, 2024, pp. 8180-8187, doi: 10.23919/CCC63176.2024.10662312.

5. Youvan, Douglas. (2024). Non-Zero-Sum Games: A Framework for Sustainable World Peace. 10.13140/RG.2.2.35331.52004.

6. Benhamida, Farid. (2006). A solution method to economic dispatch using the matlab function (FMINCON). Modelling, Measurement and Control A. 79. 1-13.

7. Gulbakyt S., Almaz A. (2024), “Development of a mathematical model for the distribution of financial resources”. Procedia Computer Science, Volume 251, Pages 520-525, ISSN 1877-0509. URL: https://doi.org/10.1016/j.procs.2024.11.142

8. Nemirovski, Arkadi & Todd, Michael. (2008). Interior-point methods for optimization. Acta Numerica. 17. 191 - 234. 10.1017/S0962492906370018.

9. Gondzio, J. (2012). Interior point methods 25 years later. European Journal of Operational Research, 218(3), 587-601. https://doi.org/10.1016/j.ejor.2011.09.017

10. Bureau of National Statistics of the Agency for Strategic Planning and Reforms of the Republic of Kazakhstan // [Electronic resource]: URL: https://stat.gov.kz/ru/region/almatyobl /. Date of application: 1.12.2024.

11. Information and legal system of regulatory legal acts of the Republic of Kazakhstan. Institute of Legislation and Legal Information, Ministry of Justice of the Republic of Kazakhstan. // [Electronic resource]: URL: https://adilet.zan.kz/rus/docs/V20D0005841.

12. Vafaei, Nazanin & Ribeiro, Rita & Camarinha-Matos, Luis. (2018). Selection of Normalization Technique for Weighted Average Multi-criteria Decision Making. 10.1007/978-3-319-78574-5_4.

13. Vafaei, N., Ribeiro, R. A., & Camarinha-Matos, L. M. (2022). Assessing normalization techniques for simple additive weighting method. Procedia Computer Science, 199, 1229-1236. https://doi.org/10.1016/j.procs.2022.01.156

14. Navarro-Gonzalez, F. & Villacampa, Yolanda. (2021). A Foundation for Logarithmic Utility Function of Money. Mathematics. 9. 10.3390/math9060665.

НӨЛДІК ЖИЫНТЫҚ ЕМЕС ОЙЫН ҮЛГІСІ АРҚЫЛЫ АЙМАҚТЫҚ БЮДЖЕТТІ БӨЛУДІҢ ОҢТАЙЛАНДЫРУЫ

Аңдатпа. *Бұл жұмыста үш қатысушының: облыс азаматтарының (дауыс беру арқылы), мәслихаттардың (өңірлерді дамытудың стратегиялық мақсаттарын есепке алу арқылы) және бизнестің (инвестицияларды қайтаруға бағытталған) мүдделерін ескеретін бюджетті бөлудің нөлден тыс ойын моделі әзірленді. Кәдімгі сызықтық модельдерден айырмашылығы, бұл әдіс мүдделі тараптардың қайшылықты мүдделерін және шешім қабылдау әрекеттерін жақсырақ түсіру үшін күрделі пайдалы функцияларды пайдаланады. Азаматтар үшін азайып бара жатқан шекті табыстарды есепке ала отырып, логарифмдік пайдалылық функциясы секторға көбірек инвестиция салу біртебірте аз қабылданатын артықшылықтарға әкелетініне көз жеткізеді.*

Модель сандық оңтайландыру әдістерін, атап айтқанда, сызықты емес шектеулерді тиімді өңдейтін және бюджеттің жаһандық теңгерілген бөлінуін қамтамасыз ететін ішкі нүкте алгоритмін пайдалана отырып жүзеге асырылады. Оңтайландыру процесі мақсат функциясын біртіндеп жақсарта отырып, 40 итерацияны қажет етті, соңғы жалпы пайдалылық 9,5681 × 10^6-ке жетті. Нәтижелер ұсынылған модельдің аймақтық басымдықтар мен экономикалық ойларға сәйкестікті сақтай отырып, ресурстарды әділ және тиімді бөлу мүмкіндігін растайды.

Бұдан басқа, зерттеу бюджетті бөлудегі ойын-теориялық модельдердің практикалық салдарын көрсетеді, мүдделі тараптар арасындағы стратегиялық өзара әрекеттестік шешімдер қабылдаудың неғұрлым ашық және ұтымды процестерін қалай қалыптастыра алатындығын көрсетеді. Нәтижелер бұл тәсілді динамикалық жоспарлау және стратегиялық модельдеу үшін кеңейтуге болатынын көрсетеді, бұл бейімделу стратегияларын, Нэш тепе-теңдік талдауын және өзгеретін әлеуметтік-экономикалық жағдайларға негізделген нақты уақыттағы түзетулерді қамтитын болашақ жақсартуларға мүмкіндік береді. Ұсынылып отырған модель бюджетті бөлудің тиімділігін, әділдігін және ашықтығын айтарлықтай арттырып, оны мемлекеттік қаржы және саясатты жоспарлау саласындағы шешімдер қабылдаушы тұлғалар үшін перспективалық құралға айналдырады.

Түйін сөздер*: бюджет, ақпараттық процесті модельдеу, оңтайландыру, ойын моделі, нөлдік емес қосынды моделі, ішкі нүкте алгоритмі, пайдалы функция, Нэш тепе-теңдігі.*

ОПТИМИЗАЦИЯ РАСПРЕДЕЛЕНИЯ РЕГИОНАЛЬНОГО БЮДЖЕТА С ПОМОЩЬЮ МОДЕЛИ ИГРЫ С НЕНУЛЕВОЙ СУММОЙ

Аннотация. *В данной статье разрабатывается модель игры с ненулевой суммой для распределения бюджета через имитационное моделированиес помощтбю программного обеспечения Matlab, которая учитывает интересы трех участников: граждан региона (через голосование), маслихатов (через учет стратегических целей регионального развития) и предприятий (ориентированных на окупаемость инвестиций). В отличие от обычных линейных моделей, этот метод использует сложные функции полезности для лучшего охвата конфликтующих интересов заинтересованных сторон и поведения при принятии решений. Фиксируя уменьшающиеся предельные выгоды для граждан, логарифмическая функция полезности гарантирует, что больше инвестиций в сектор приводит к постепенному уменьшению воспринимаемых преимуществ.*

Модель реализована с использованием методов численной оптимизации, в частности алгоритма внутренней точки, который эффективно обрабатывает нелинейные *ограничения и обеспечивает глобально сбалансированное распределение бюджета. Процесс оптимизации потребовал 40 итераций, постепенно улучшая целевую функцию, при этом окончательная общая полезность достигла 9,5681 × 10⁶. Результаты подтверждают способность предлагаемой модели распределять ресурсы справедливо и эффективно, сохраняя соответствие региональным приоритетам и экономическим соображениям.*

Исследование подчеркивает практические результаты игровых теоретико-моделей в распределении бюджета, демонстрируя, как стратегические взаимодействия между заинтересованными сторонами могут формировать более прозрачные и рациональные процессы принятия решений. Результаты показывают, что этот подход может быть расширен для динамического планирования и стратегического моделирования, что позволяет в будущем усовершенствовать его, включив адаптивные стратегии, анализ равновесия Нэша и корректировки в реальном времени на основе меняющихся социально-экономических условий. Предложенная модель значительно повышает эффективность, справедливость и прозрачность распределения бюджета, что делает ее перспективным инструментом для лиц, принимающих решения в области государственных финансов и планирования политики.

Ключевые слова: *бюджет, моделирование информационных процессов, оптимизация, игровая модель, модель с ненулевой суммой, алгоритм внутренней точки, функция полезности, равновесие Нэша.*

Сведение об авторах

Абдуалиев Алмаз Едігеұлы	PhD докторант, магистр технических наук, сеньор-лектор кафедры «Информационные системы», Международный Университет Информационных Технологий, Алматы, Казахстан, a.abdualiyev@iitu.edu.kz, https://orcid.org/0009-0009-5950-8232;
Ибрагим Гүлнұр Қуандыққызы	Магистр технических наук, преподаватель кафедры “КОиХИ”, Казахский национальный исследовательский технический университет имени К. Сатпаева, Алматы, Казахстан, g.ibragim@satbayev.university, https://orcid.org/0000-0003-0974-106X
Сембина Гулбакыт Какеевна	Кандидат технических наук, ассоц.профессор кафедры «Информационные системы», Международный Университет Информационных Технологий, Алматы, Казахстан, g.sembina@iitu.edu.kz, https://orcid.org/0000-0003-2920-1490;
Изханова Ботагоз Егембердіқызы	Магистр технических наук, старший преподаватель кафедры «Иформационно-Коммуникационные Технологий», АО Университет имени Жумабека Ахметовича Ташенева, Шымкент, Казахстан, dellivine@mail.ru, https://orcid.org/0009-0008-0688-6224;

Авторлар туралы мәлімет

Абдуалиев Алмаз Едігеұлы	PhD докторант, техника ғылымдарының магистрі, Халықаралық Ақпараттық Технологиялар Университетінің «Ақпараттық жүйелер» кафедрасының сеньор-лекторы, Алматы, Қазақстан, a.abdualiyev@iitu.edu.kz, https://orcid.org/0009-0009-5950-8232;
Ибрагим Гүлнұр Қуандыққызы	Техника ғылымдарының магистрі, «АКӨжС» кафедрасының оқытушысы, Қ.Сәтбаев атындағы Қазақ Ұлттық Ғылыми-Зерттеу Техникалық Университеті, Алматы, Қазақстан, g.ibragim@satbayev.university, https://orcid.org/0000-0003-0974-106X

Сембина Гүлбақыт Какейқызы	Техника ғылымдарының кандидаты, Халықаралық Ақпараттық Технологиялар Университетінің «Ақпараттық жүйелер» кафедрасының доценті, Алматы, Қазақстан, g.sembina@iitu.edu.kz, https://orcid.org/0000-0003-2920-1490;
Изханова Ботагоз Егембердіқызы	Техника ғылымдарының магистрі, «Жұмабек Ахметұлы Ташенев атындағы университет» АҚ «Ақпараттық-коммуникациялық технологиялар» кафедрасының аға оқытушысы, Шымкент, Қазақстан, dellivine@mail.ru, https://orcid.org/0009-0008-0688-6224;

Information about the authors

Abdualiyev Almaz Edigeuly	Master of technical sciences, Senior - Lecturer of the Information Systems Department, International Information Technologies University, Almaty, Kazakhstan, a.abdualiyev@iitu.edu.kz, https://orcid.org/0009-0009-5950-8232
Ibrahim Gulnur Kuandykovna	Master of technical sciences, Lecturer of Department of Cybersecurity, Information Processing and Storage, Kazakh National Research Technical University named after K. Satbayev, Almaty, Kazakhstan, g.ibragim@satbayev.university, https://orcid.org/0000-0003-0974-106X
Sembina Gulbakhyt Kakeyovna	Candidate of technical sciences, Associate Professor of the Information Systems Department, International Information Technologies University, Almaty, Kazakhstan, g.sembina@iitu.edu.kz, https://orcid.org/0000-0003-2920-1490
Izkhanova Botagoz Egemberdiyevna	Master of technical sciences, Senior - Lecturer, Department of Information and Communication Technologies, JSC University named after Zhumabek Akhmetovich Tashenev, Shymkent, Kazakhstan, dellivine@mail.ru, https://orcid.org/0009-0008-0688-6224